

Thursday, 20 Feb.

Start w/ 114 field redefinition
=> then add chiral fermions

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Suppose we have a complex scalar coupled to fermions with a Chiral $U(1)$ symmetry

$$\mathcal{L} = \bar{\psi}_L i\cancel{D} \psi_L + \bar{\psi}_R i\cancel{D} \psi_R + |\partial_\mu \phi|^2$$

$$- h \bar{\psi}_L \psi_R \phi - h^* \bar{\psi}_R \psi_L \phi^\dagger$$

$$- V(\phi)$$

$$V(\phi) = \mu^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2$$

$$\phi \neq \phi_R$$

We can take $h = h^*$ w/o loss

$\psi_L' = e^{i(\arg h)} \psi_L$ to absorb any complex phase of h
This is not a symmetry

This \mathcal{L} is invariant under the Chiral transformation

$$\phi \rightarrow e^{i\beta} \phi, \quad \psi_R \rightarrow e^{-i\beta} \psi_R, \quad \psi_L \rightarrow \psi_L$$

Notice the freedom in choosing this chiral transformation. This is not a unique choice, but all have the same consequences

Notice this symmetry prohibits a fermion mass term

$$\bar{\psi}_L \psi_R \rightarrow e^{-i\beta} \bar{\psi}_L \psi_R \neq \bar{\psi}_L \psi_R$$

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Let us work in the same Hermitian basis, and let

$$\mu^2 < 0$$

This drives spontaneous symmetry breaking. Let us again chose the direction

$$\langle \phi \rangle = \begin{pmatrix} v \\ 0 \end{pmatrix} \quad \text{in the basis } \phi = \begin{pmatrix} \sigma \\ \pi \end{pmatrix}$$

Let us then perturb about vacuum

$$\sigma = \sigma' + v$$

$$\mathcal{L}_{\text{ Yukawa}} = -h \bar{\psi}_L \gamma_R \phi - h \bar{\psi}_R \gamma_L \phi^+$$

$$\Rightarrow -h \left[\bar{\psi}_L \gamma_R (v + \sigma' + i\pi) + \bar{\psi}_R \gamma_L (v + \sigma' - i\pi) \right]$$

$$= -h \underbrace{[\bar{\psi}_L \gamma_R + \bar{\psi}_R \gamma_L]}_{\bar{\psi} \psi} (v + \sigma') - h \underbrace{[\bar{\psi}_L \gamma_R - \bar{\psi}_R \gamma_L]}_{\bar{\psi} \gamma_5 \psi} i\pi$$

$$\bar{\psi} \psi$$

$$\bar{\psi} \gamma_5 \psi$$

$$= -hv \bar{\psi} \psi - h \bar{\psi} \psi \sigma' - h \bar{\psi} \gamma_5 \psi i\pi$$

Notice, there is now a fermion mass term!

The fermions are now coupled to both a scalar of mass

$$m_\sigma = \sqrt{2}\mu$$

and a mass less pseudo-scalar π

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Let's now couple chiral fermions to this complex scalar

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In the Polar basis, the scalar \mathcal{L} is the same, and the Yukawa interactions are

$$\mathcal{L}_{\text{Yuk}} = -\frac{h v}{\sqrt{2}} \bar{\psi}_L \psi_R \left(1 + \frac{P}{v} \right) e^{i\theta/v} + \text{h.c.}$$

so we see again the presence of terms of all orders in (θ/v) , a manifestation of the EFT.

However, we can exploit the $U(1)$ symmetry to remove this phase $\psi'_R = e^{i\theta/v} \psi_R$

$$\Rightarrow \mathcal{L}_{\psi} = \bar{\psi}_L i\cancel{D} \psi_L + \bar{\psi}_R i\cancel{D} \psi_R + \frac{\partial_v \theta}{v} \bar{\psi}_R \gamma^5 \psi_R$$

$$- \frac{h v}{\sqrt{2}} \bar{\psi}_L \psi_R \left(1 + \frac{P}{v} \right)$$

Notice 2 important things

1) the θ is now derivatively coupled to ψ

2) the original \mathcal{L} prohibits an explicit mass term for the fermions $\bar{\psi}_L \psi_R \rightarrow \bar{\psi}_L \psi_R e^{iP} \neq \bar{\psi}_L \psi_R$

However, after SSB, the fermions have a mass term! proportional to the Yukawa coupling ϵ the v.v.

SSB provides a natural mechanism to give rise to fermions

So what have we learned?

- SSB gives rise to massless modes naturally
- the masses of the GB are protected from renormalization
- SSB can give mass to other fields, as $\vec{p} \Rightarrow 0$, their interactions must vanish
- SSB can give rise to fermion mass terms when otherwise symmetry prohibits such a term
 - this is how the fermions get their masses in SM (Higgs Mechanism)
 - this is how the proton gets its mass (QCD)

Questions?

- what if we promote global symmetry to local gauge symmetry? Higgs Mechanism
- what if we add an explicit symmetry breaking term?

How does this work for QCD?

Consider $SU(2)$ flavor, $\psi = \begin{pmatrix} u \\ d \end{pmatrix}$

$$\mathcal{L} = \bar{\psi} [i\cancel{D} - m] \psi$$

Lets first focus
on isospin limit

$$m_u = m_d \Rightarrow m = \begin{pmatrix} u & u \\ d & d \end{pmatrix} = S_{ij} m$$

$$\mathcal{L} = \bar{\psi}^i_a [i \gamma^\mu [\partial_\mu \delta_{ab} \delta_{ij} + ig(A_\mu)_{ab}^b \delta_{ij}] - m \delta_{ij} \partial_\mu] \psi_{jb}$$

$D_{\mu,ab} \cdot \delta_{ij}$

(at this time, upper/lower
index only matter for
Lorentz)

$$\mathcal{L} = \bar{\psi}_L i\cancel{D} \psi_L + \bar{\psi}_R i\cancel{D} \psi_R - \bar{\psi}_L m \psi_R - \bar{\psi}_R m \psi_L$$

~~Let us~~ This theory is invariant under the

Let us first ignore the mass term.

The resulting theory is invariant under a global

$$U(2)_L \otimes U(2)_R$$

flavor-chiral symmetry.

If this symmetry were realized in nature, we would expect parity partners which were degenerate

$$\Psi_L = \frac{1 - \gamma_5}{2} \psi$$

$$P \Psi_L = \gamma^0 \left(\frac{1 - \gamma_5}{2} \right) \psi(-x)$$

$$= \frac{1 + \gamma_5}{2} \psi(-x)$$

$$= \Psi_R(-x)$$

Parity, which is also a symmetry of the theory, transforms $L \leftrightarrow R$.

So the nucleon parity even \neq parity odd states should be (nearly) degenerate

$$m_N \approx 939 \text{ MeV} \quad m_{N^-} \approx 1535 \text{ MeV}$$

The parity partners are not close to degenerate.

But we also observe there are 3 states, whose masses, are much less than the typical QCD scale

$$m_{\pi^\pm} \approx 139.6 \text{ MeV}$$

$$m_{\pi^0} \approx 135.0 \text{ MeV}$$

parity odd
scalar bosons

This led to the postulate that the vacuum of

QCD is not invariant under this global chiral symmetry.

$$\psi_L \rightarrow e^{i\theta_L^a t^a} \psi_L$$

$$t_a = \left\{ 1, \frac{\tau^a}{2} \right\}, \tau^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\psi_R \rightarrow e^{i\theta_R^a t^a} \psi_R$$

$$\tau^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$\tau^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

One could parameterize these transformations in terms of $\theta_V^a = \theta_L^a + \theta_R^a$ vector transformation

$$\theta_A^a = \theta_L^a - \theta_R^a \quad \text{axial-vector transformation}$$

V: $\psi \rightarrow e^{i\theta_V^a t^a} \psi$

The vector subgroup of $U(2)_L \otimes U(2)_R$
is a closed Lie algebra

A: $\psi \rightarrow e^{i\theta_A \cdot \gamma_5 t^a} \psi$

The axial vector subgroup
is not closed, ~~and so is not~~
~~properly~~

This parameterization is useful, because we observe in nature the pions are pseudo-scalars, which means they are the pseudo-goldstone-bosons of broken axial-generators.

A more accurate way to describe the postulate

$$U(2)_L \otimes U(2)_R \xrightarrow{\text{vacuum}} U(2)_V$$

What do we then expect?

We began with 8 generators of the symmetry

$$U(2) : \begin{matrix} 1 & U(1) \\ 3 & SU(2) \end{matrix}$$

$U(2)_L \otimes U(2)_R$ has 8 symmetry generators

We postulate the vacuum spontaneously breaks to the $U(1)_V$ subgroup.

which means we are left with 4 generators, so 4 correspond to transformations which ~~do not~~ longer leave the vacuum invariant.

So we expect 4 (nearly) massless modes.

But we only observe 3.

- The other important piece to this story is that in the full quantum theory, the $U(1)_L \otimes U(1)_R$ is not the symmetry group. Only $U(1)_V$ is a symmetry, the $U(1)_A$ transformation is "anomalous".

So really, we began with 7 generators, are left with 4, and so expect 3 (nearly) massless modes.

- We also have the postulate of confinement, that quarks are bound.

- So we further postulate that the spontaneous symmetry breaking is driven by

$$\langle \Omega | \bar{q}_i q_j | \Omega \rangle = \lambda \delta_{ij} \neq 0$$

$$\langle \Omega | \bar{q}_i^R q_i^L | \Omega \rangle = \lambda S^{ij} \neq 0$$

$$\downarrow \quad \text{SU}(2)_L \otimes \text{SU}(2)_R$$

$$\langle \Omega | \bar{q}_k^R R_k^+ L_\ell^\dagger q_\ell^L | \Omega \rangle \quad L = e^{i\theta_L^\alpha t^\alpha} \\ R = e^{i\theta_R^\alpha t^\alpha}$$

$$= \langle \Omega | \bar{q}_k^R q_\ell^L | \Omega \rangle \cdot R_k^+ L_\ell^\dagger$$

$$= \lambda S_{kl} R_k^+ L_\ell^\dagger$$

$$= \lambda (R^+ L)^{ij}$$

$$= \lambda \sum ij$$

$$\text{If } \theta_R^\alpha = \theta_L^\alpha \Rightarrow (R^+ L)^{ij} = \delta^{ij}$$

Otherwise, the vacuum state is not invariant

Under the global chiral transformation

$$\Sigma \rightarrow L \Sigma R^+$$

This Σ field should parameterize our system.

Motivated by our discussion of SSB

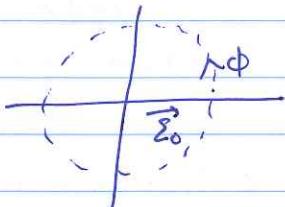
$$\mathcal{L} = \Sigma_0 e^{\frac{2i\phi}{f}}$$

f is the v-ev., and we will see, is also related to the pion decay constant, measured in the weak decay of the pion

Σ_0 is the "radial" excitation, ~~where in our case, the vector transformations~~

ϕ are the "angular" excitations

But in our more interesting group space.



$$\phi = \begin{pmatrix} \pi^0/\sqrt{2} & \pi^+ \\ \pi^- & -\pi^0/\sqrt{2} \end{pmatrix} = \sum_i t_i \pi_i \sqrt{2}, \quad t_i = \frac{\gamma_i}{2}$$

So, we see we can use a non-linear realization of the chiral symmetry to parameterize the pion fields.

So we can build a \mathcal{L} out of these fields.
What are the options?

- 1) ignore radial excitations.

We discussed last time, the notion of EFT.

We will restrict our interest to energies where the Σ_0 can be considered as heavy and integrated out

$$\frac{i}{p^2 - M^2} \rightarrow \frac{-i}{M^2} + O(P^2/M^4)$$

In addition to Σ , we have Σ^+

$$\Sigma \rightarrow L \Sigma R^\dagger$$

$$\Sigma^+ \rightarrow R \Sigma^+ L^\dagger$$

Where Σ came from considering $\langle \Sigma | \bar{q}_R q_L | \Sigma \rangle$
 Σ^+ comes from $\langle \Sigma | \bar{q}_L q_R | \Sigma \rangle$

$$\Sigma^+ \Sigma = 1 \quad \begin{cases} \text{(special to } SU(2), \text{ because we can exponentiate)} \\ \text{pauli-matrices} \end{cases}$$

To construct \mathcal{L} , we need operators which respect all symmetries of QCD, and are real

$$\text{Tr}(\Sigma \Sigma^+) = \text{Tr}(1) \quad \text{not interesting}$$

$$\text{Tr}(\partial_\mu \Sigma \partial^\mu \Sigma^+)$$

$$\downarrow \text{SU}(2)_{L \times R}$$

$$\partial_\mu (L \Sigma R^\dagger) \partial^\mu (R \Sigma^+ L^\dagger)$$

$$= L \partial_\mu \Sigma \partial^\mu \Sigma^+ L^\dagger$$

so under trace, this term is invariant under chiral transformations

$$\mathcal{L} \supset c_2 \text{Tr}(\partial_\mu \Sigma \partial^\mu \Sigma^+)$$

$$\partial_\mu \Sigma \partial^\mu \Sigma ?$$

$$\downarrow \text{SU}(2)_{L \times R}$$

$$\partial_\mu \Sigma L \Sigma R^\dagger \partial^\mu L \Sigma^+ R^\dagger$$

$$= L \partial_\mu \Sigma R^\dagger L \partial^\mu \Sigma R^\dagger$$

under trace, only invariant under vector ($L=R$) subgroup

$$\mathcal{L} \partial_\mu \Sigma^\dagger$$

This term is invariant, but it
is exactly related to
 $\partial_\mu \Sigma \partial^\mu \Sigma^\dagger$
so is not a new operator.

At $O(\mathcal{L}^4)$, there is a unique operator

$$\mathcal{L} = c_2 \text{Tr} (\partial_\mu \mathcal{L} \partial^\mu \mathcal{L}^\dagger)$$

c_2 = coefficient of the operator. While the symmetry tells us the form of the operators, it can not fix the coefficients.

In theories where the EFT and underlying fundamental theory can be matched, (when fund. theory can be handled with perturbation theory) we can determine the coefficients by matching S-matrix elements computed in both EFT and fund. theory.

In general, these coefficients are known as "low-energy-constants" as they parameterize short-distance physics at low energies.

In theories like QCD, where non-perturbative dynamics dominate the interactions, we can not compute these ~~↪~~ LECs directly. We can determine them phenomenologically by comparing predictions with physical observables
(or Lattice QCD calculations)

Let us proceed

$$\mathcal{L} = C_2 \text{Tr} (\partial_\mu \Sigma \partial^\mu \Sigma^+) \quad \det \Sigma = 1$$

$$\Sigma = e^{2i\phi/f}, \quad \Sigma^+ = e^{-2i\phi/f} \quad (\downarrow \phi^+ = \phi) \\ \text{Tr } \phi = 0$$

$$\Sigma = 1 + \frac{2i\phi}{f} + \frac{1}{2} \left(\frac{2i\phi}{f} \right)^2 + \dots$$

$$\Sigma^+ = 1 - \frac{2i\phi}{f} + \frac{1}{2} \left(-\frac{2i\phi}{f} \right)^2 + \dots$$

$$\mathcal{L} = C_2 \text{Tr} \left[\partial_\mu \left(1 + \frac{2i\phi}{f} + \frac{1}{2} \left(\frac{2i\phi}{f} \right)^2 + \dots \right) \partial^\mu \left(1 - \frac{2i\phi}{f} + \left(\frac{-2i\phi}{f} \right)^2 + \dots \right) \right]$$

$$= C_2 \text{Tr} \left[\partial_\mu \frac{2i\phi}{f} \partial^\mu \frac{-2i\phi}{f} + \partial_\mu \frac{2i\phi}{f} \partial^\mu \left(\frac{2i\phi}{f} \right)^2 + \frac{1}{2} \partial_\mu \left(\frac{2i\phi}{f} \right)^2 \partial^\mu \left(\frac{2i\phi}{f} \right) + \dots \right]$$

$$= C_2 \text{Tr} \left[\frac{4}{f^2} \partial_\mu \phi \partial^\mu \phi + O(\partial^2, \phi^4/f^4) \right] \quad \text{Tr} [\phi^{\text{odd}-n}] = 0$$

$$= \frac{4C_2}{f^2} \text{Tr} (\partial_\mu \phi \partial^\mu \phi), \quad \phi = \begin{pmatrix} \pi^0/\sqrt{2} & \pi^+ \\ \pi^- & -\pi^0/\sqrt{2} \end{pmatrix}$$

$$= \frac{4C_2}{f^2} \cdot \left[2 \partial \pi^+ \partial \pi^- + \partial \pi^0 \partial \pi^0 \right]$$

$$= \frac{8C_2}{f^2} \left(|\partial \pi^+| + \frac{1}{2} \partial \pi^0 \partial \pi^0 \right)$$

So, canonical normalization of fields requires

$$C_2 = \frac{f^2}{8}$$

$$\mathcal{L} = \frac{f^2}{8} \text{Tr} (\partial_\mu \Sigma \partial^\mu \Sigma^+)$$